Models and Languages for Computational Systems Biology

Lecture 4: Labelled Transition Systems

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Review: Matrices

Every Petri Net can be precisely represented by two matrices:

- the *pre-state* matrix Pre, also known as *flow-in* F_{IN};
- the *post-state* matrix Post or *flow-out* $\mathrm{F}_{\mathrm{OUT}}$.

Both markings and (multi)transitions are then represented by vectors.

The difference between these $\mathrm{A} = (\mathrm{Post} - \mathrm{Pre})$ is the *activity*, *reaction*, *stoichiometry*, or *incidence* matrix of the net.

This identifies the change to a marking which happens on the firing of a multiset of transitions.

Review: Invariants

From the incidence matrix, we can identify some distinctive vectors.

- A *transition invariant* is a multitransition that causes no change in the marking.
- A *place invarariant* is a weighted sum of markings that is unchanged by any transition.
- A *minimal* transition or place invariant is one that cannot be expressed as an integer linear combination of others.

The numbers of places, transitions and independent invariants are all related:

$$(\#\text{T-invariants}) - (\#\text{P-invariants}) \;=\; (\#\text{Transitions}) - (\#\text{Places})$$

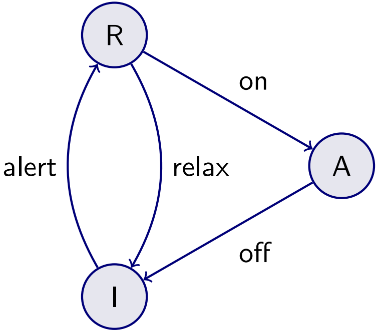
The P- and T-invariants span the *null space* of the incidence matrix

Labelled Transition Systems

A *labelled transition system* (LTS) comprises some number of states, with arcs between them labelled by activities of the system.

Labelled transition systems are suitable for modelling discrete-state systems that change through actions of some kind.

Certain states may be distinguished: a start state, perhaps one or more final states.



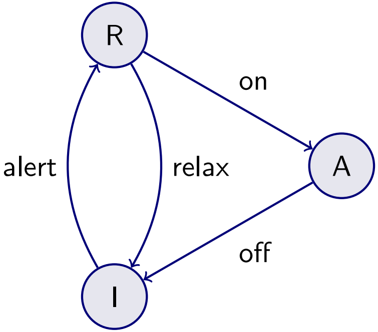
Labelled Transition Systems

A labelled transition system is specified by:

- A set S of *states*;
- A set L of *labels* or *actions*;
- A set of *transitions* $T \subseteq S \times L \times S$.

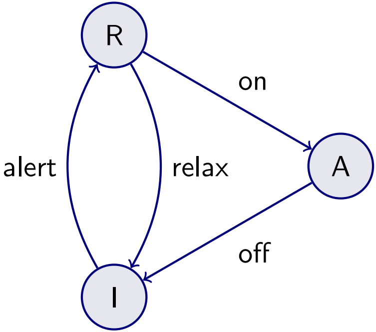
Transitions are given as triples $(\text{start}, \text{label}, \text{end})$. The set of states

may be finite or infinite; the set of labels is usually finite.



Labelled Transition Systems

$$S = \{\mathsf{I}, \mathsf{R}, \mathsf{A}\}$$
$$\mathrm{L} = \{\mathsf{alert}, \mathsf{relax}, \mathsf{on}, \mathsf{off}\}$$
$$\mathrm{T} = \{(\mathsf{I}, \mathsf{alert}, \mathsf{R}), (\mathsf{R}, \mathsf{relax}, \mathsf{I}), (\mathsf{R}, \mathsf{on}, \mathsf{A}), (\mathsf{A}, \mathsf{off}, \mathsf{I})\}$$



Labelled Transition Systems

A *run* of a labelled transition system is a list of transitions that proceed from one state to another:

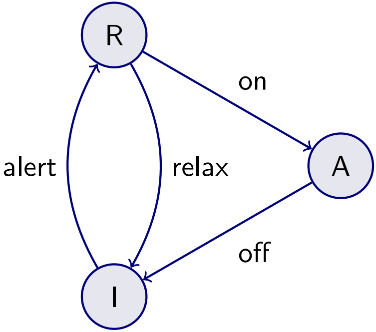
$$(s_1, l_1, s_2), (s_2, l_2, s_3), \ldots, (s_k, l_k, s_{k+1})$$

The *trace* of a run is the series of labels from these transitions:

$$l_1 l_2 l_3 \ldots l_k$$

Both runs and traces may be finite or infinite.

For any given system, we are generally interested in the set of all traces from a given initial state.



Labelled Transition Systems

For example, all these are traces of the LTS on the right:

on.off

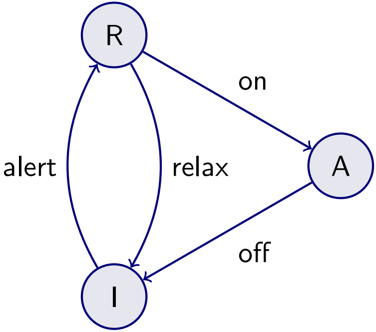
alert.on.off

alert.relax.alert.on.off.alert

ε (empty trace)

This is a finite LTS, but its complete set of traces is infinite.

The set of traces for an LTS gives some information about the behaviour of the system: but it is not enough to reconstruct the LTS itself.

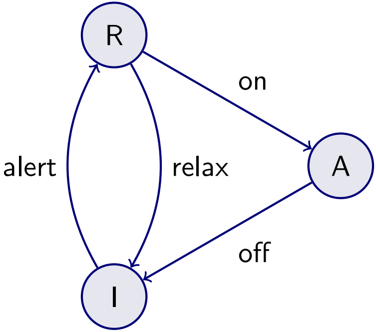


Labelled Transition Systems

Labelled transitions systems, runs and traces are closely related to a range of similar notions in computer science:

- Finite-state machines / Automata
- Regular languages and regular expressions
- Moore and Mealy machines

In the next lectures we shall look at logics for describing properties of traces, and methods to determine whether they are satisfied by some, all, or none of the traces for a labelled transition system.



From Petri Net to LTS

From any Petri Net we can generate a labelled transition system that represents its behaviour.

- The states of the LTS are all possible markings of the net.
- The labels of the LTS are the transitions of the net.
- The transitions of the LTS are all the firings of the net.

If $M \xrightarrow{T} N$ is a firing of transition T in a net, taking marking M to marking N, then there is a corresponding transition (M, T, N) of the LTS.

We could instead use multitransitions as labels, but it is more usual to have transitions of the LTS correspond to single-transition firings of the Petri Net.

Reachability Graph

Given a specific initial marking of a net, we can construct the labelled transition system of all reachable markings. This is its *reachability graph*.

The full LTS for a Petri Net will be infinite; however, the reachability graph from a particular initial marking may be finite.

The LTS from a Petri Net expresses some of its behaviour, but not all: it is not possible to uniquely reconstruct a Petri Net from its transition system.

Homework

Exercises

Questions from previous MLCSB exam papers:

- 2007 Question 1
- 2008 Questions 1(a)–(e) and 3(a)–(b)

Please bring written answers to next Thursday's lecture. I shall mark these, but they are not for credit.

Preparation

Next time we shall be looking at languages for describing properties of traces, in particular *Linear Temporal Logic* (LTL).

It may help to revise *propositional logic* before then.