Models and Languages for Computational Systems Biology

Lecture 5: Linear Temporal Logic

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Labelled Transition Systems

A *labelled transition system* (LTS) comprises states joined by arcs labelled with activities.

A *run* of an LTS is a list of transitions leading from one state to another:

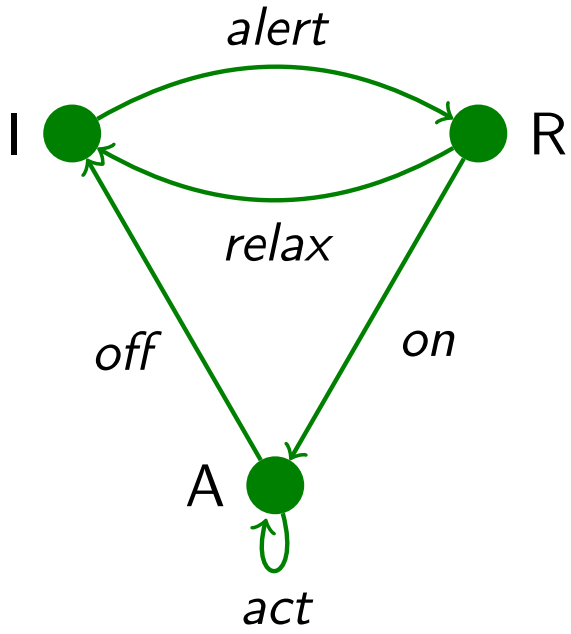
$$\mathrm{I} \xrightarrow{\text{alert}} \mathrm{R} \xrightarrow{\text{on}} \mathrm{A} \xrightarrow{\text{act}} \mathrm{A}$$

The *trace* of a run is its transition labels:

alert.on.act

Runs and traces may be finite or infinite.

The traces for an LTS give information about the behaviour of the system: but are not enough to reconstruct the LTS itself.



Petri Net to LTS

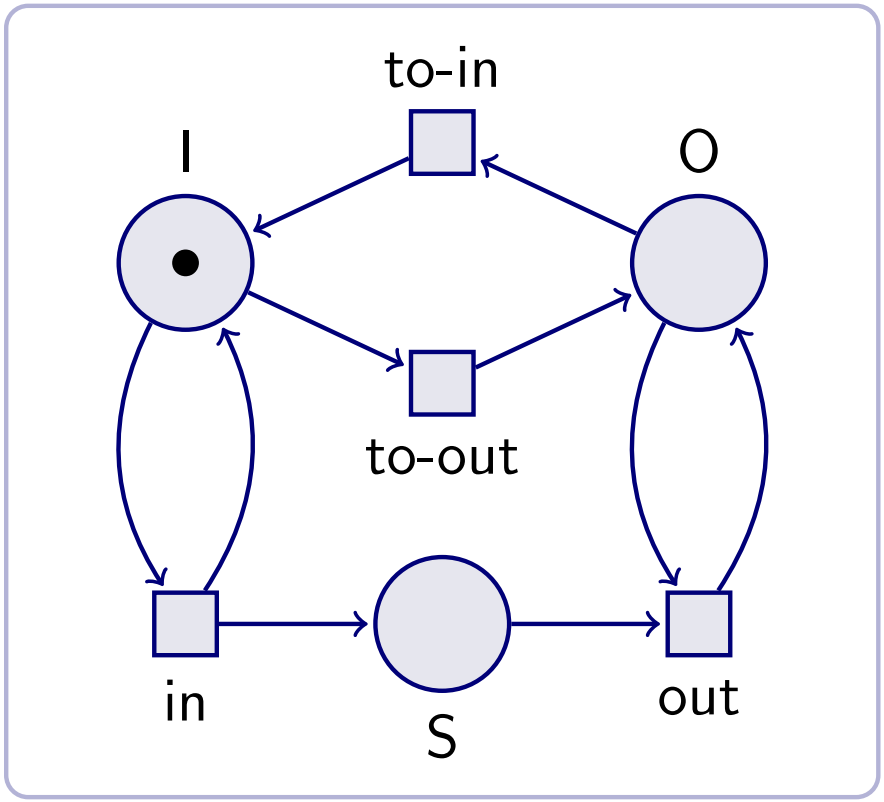
From any Petri net we can obtain a labelled transition system representing its behaviour.

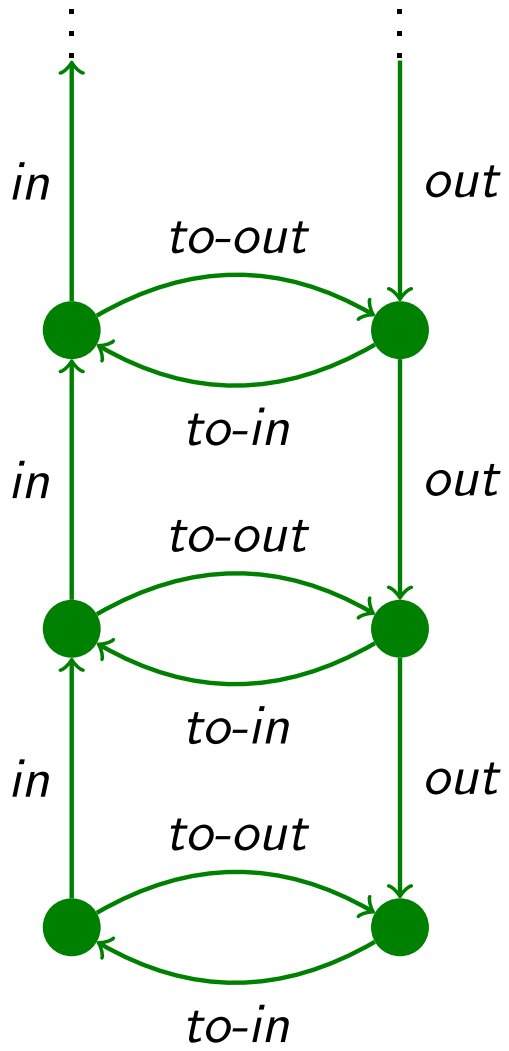
- States of the LTS are markings of the net.
- Transitions of the LTS are firings of the net.

We can observe some properties of the LTS for a net.

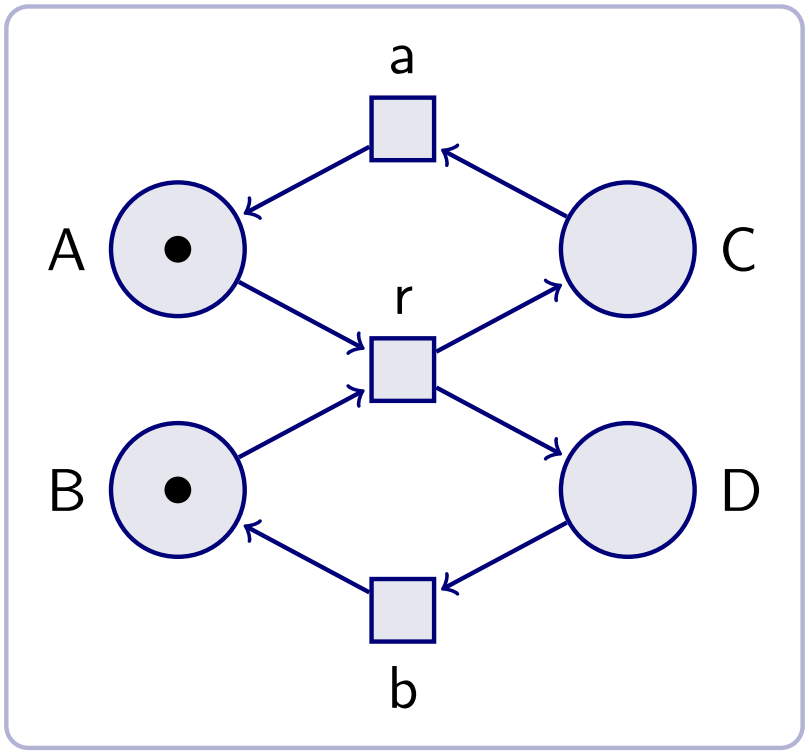
- The LTS will have an infinite number of states.
- The *reachability graph* for a net with a given initial marking may be finite.
- The traces for possible runs of the LTS can be read directly from the Petri net.
- Some information is lost: it is not possible to recover the Petri net from the LTS alone.

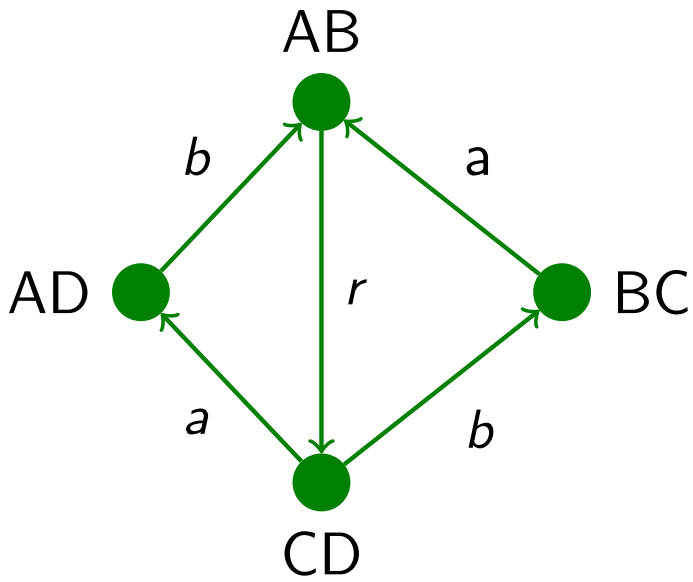
Shepherded Transport





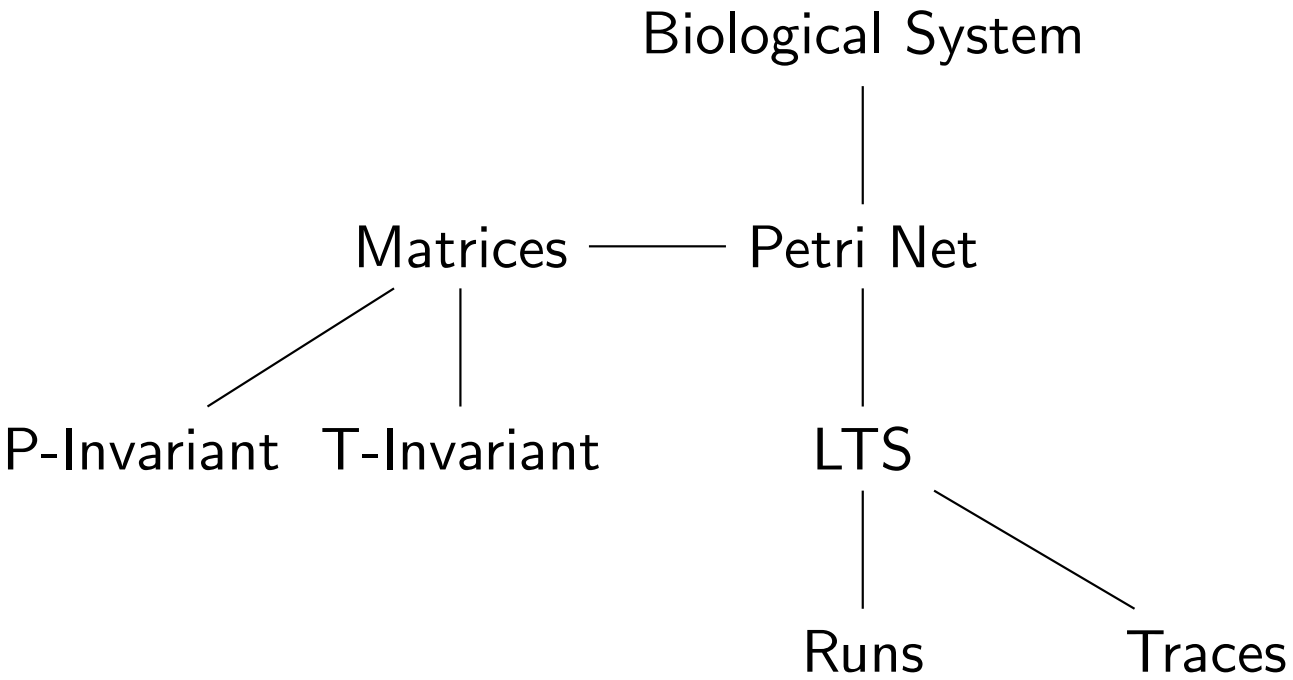
Repeating Reaction





Exercise: Find the reachability graph for #A=2, #B=1

Layers of Interpretation



Logics and Model-Checking

A *logic* is a formal language for describing some assertions.

A particular logical assertion may be evaluated in a specific situation; where it may or may not turn out to hold.

The algorithmic evaluation of logical statements at particular states in some behavioural structure is known as *model checking*.

Linear Temporal Logic

Linear Temporal Logic (LTL) describes properties of system runs.

LTL is a *logic*: a formal language for presenting assertions.

LTL is *temporal*: it speaks about behaviour over time.

LTL is *linear*: it only addresses properties of a single run.

The counterpart to linear-time is *branching-time*, which we shall see later.