Models and Languages for Computational Systems Biology

Lecture 6: Branching Time and CTL

Ian Stark

School of Informatics
The University of Edinburgh

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Linear Temporal Logic

$$\mathrm{P} ::= \mathit{Atom} \mid \mathrm{P} \wedge \mathrm{Q} \mid \mathrm{P} \vee \mathrm{Q} \mid \mathrm{P} \Rightarrow \mathrm{Q} \mid \neg \mathrm{P}$$
$$\mathbf{X}\mathrm{P} \mid \mathbf{F}\mathrm{P} \mid \mathbf{G}\mathrm{P} \mid \mathrm{P}\,\mathbf{U}\,\mathrm{Q} \mid \mathrm{P}\,\mathbf{R}\,\mathrm{Q}$$

Atom	Some basic predicate over system states
$\mathbf{X}\mathrm{P}$	Next: P holds after a single step
$\mathbf{F}\mathrm{P}$	Finally/Future: P holds at some later state
$\mathbf{G}\mathrm{P}$	Globally: P holds at this and all later states
$\mathrm{P}\,\mathbf{U}\,\mathrm{Q}$	Until: Q holds in the future, and P holds until then
$\mathrm{P}\,\mathbf{R}\,\mathrm{Q}$	Release: Q holds indefinitely, or until P also holds

An formula P of LTL is an assertion about a single *run* of a labelled transition system.

Linear Temporal Logic

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This interpretation may be extended to all runs from some given start state.

Linear Temporal Logic

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$$\mathbf{X}P \mid aP \mid \mathbf{F}P \mid \mathbf{G}P \mid P\,\mathbf{U}\,Q \mid P\,\mathbf{R}\,Q$$

Atom	Some basic predicate over system states
$\mathbf{X}P$	Next: P holds after a single step
aP	Transition: next step is a, after which P holds
$\mathbf{F}P$	Finally/Future: P holds at some later state
$\mathbf{G}P$	Globally: P holds at this and all later states
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We can add additional forms, such as a modality for each transition.

Linear Temporal Logic

$$P ::= \mathit{Atom} \mid P \wedge Q \mid P \vee Q \mid P \Rightarrow Q \mid \neg P$$
$$\mathbf{X}P \mid aP \mid \mathbf{F}P \mid \mathbf{G}P \mid P\,\mathbf{U}\,Q \mid P\,\mathbf{R}\,Q$$

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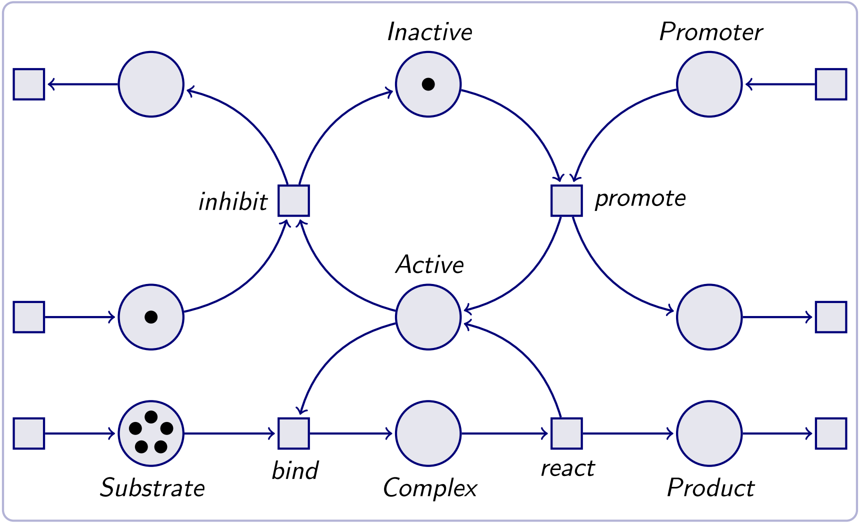
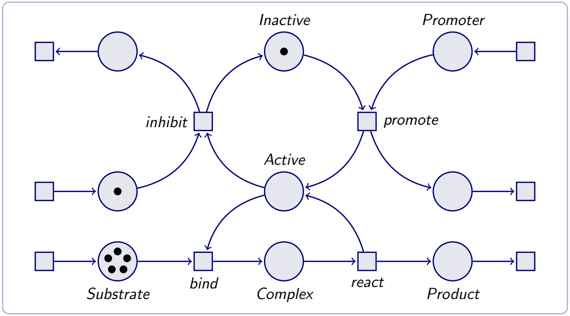
The act of checking whether a formula holds of a system is *model-checking*.

LTL Examples



LTL Examples

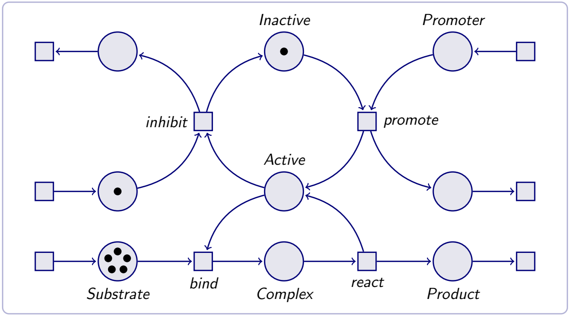


$\mathbf{X}(\mathit{Active})$

$\mathbf{F}(\mathit{Active})$

$\mathbf{G}(\mathit{Inactive} \wedge \mathit{Promoter} \Rightarrow \mathbf{X}((\neg \mathit{Promoter}) \Rightarrow \mathit{Active}))$

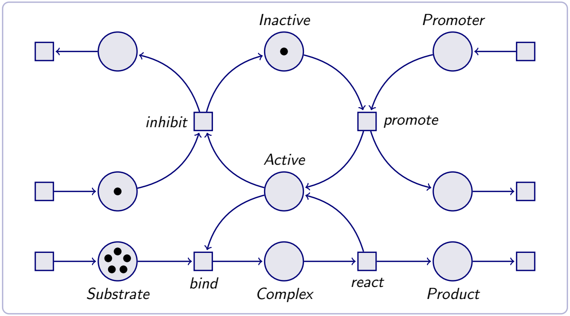
LTL Examples



$$promote.Active$$

$$\neg(promote.(\neg Active))$$

LTL Examples



$\mathbf{GF}(\mathit{Active})$

$\mathbf{F}(\mathit{Product}) \Rightarrow \mathbf{F}(\mathit{Active}))$

$\mathbf{G}(\mathit{Substrate} \wedge \mathbf{F}(\neg \mathit{Substrate}) \Rightarrow \mathbf{F}(\mathit{Active}))$

$\mathbf{G}(\mathit{Substrate} \Rightarrow (\mathit{Substrate}\,\mathbf{U}\,\mathit{Active}))$

$\mathbf{G}(\mathit{Substrate} \Rightarrow (\mathit{Active}\,\mathbf{R}\,\mathit{Substrate}))$

Further Reading

M. Heiner, D. Gilbert, and R. Donaldson.
Petri Nets for systems and synthetic biology.
In *Formal Methods for Computational Systems Biology*, LNCS 5016, pp. 215–264. Springer-Verlag, 2008.
Section 4.2(6) Model Checking of Special Behavioural Properties.

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Temporal logic patterns for querying dynamic models of cellular interaction networks.
Bioinformatics, 24(16):227–233, 2008.

F. Fages and A. Rizk.
On the analysis of numerical data time series in temporal logic.
In *Computational Methods in Systems Biology: Proceedings of the International Conference CMSB 2007*, LNBI 4695, pp. 48–63. Springer-Verlag, 2007.