Modelling residential smart energy schemes

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***Abstract*—This paper considers how a smart energy solution for residential areas can be modelled in stochastic HYPE, a process algebra that describes instantaneous, discrete stochastic and continuous deterministic behaviour. The system involves PHEVs (plug-in hybrid electric vehicles) which have batteries that can be charged either from the grid or from wind turbines, and can be viewed as a collective adaptive system (CAS). With a language such as stochastic HYPE, easy experimentation with the model and exploration of modifications of the basic scenario are possible. However, simulation can be infeasible for more complex models or larger models, and the paper discusses future work involving abstractions of the model that mitigate this problem.**

I. Introduction

Traditionally, the production of electrical energy has been centralised (for economies of scale) at power plants of various types with delivery of this energy to consumers through a distribution network involving transformers that appropriately modify the voltage [1]. This is often referred to as the grid, and can be seen as the flow of electricity from a few generation points to a large number of consumers. New technologies and the desire to use renewable sources of energy or reduce energy consumption have led to more distributed generation and the need for information to allow the immediate management of energy consumption and storage [1], [2]. The term *smart grid* is used to describe the system of electricity generation and consumption that requires both information and electricity to flow in both directions between generators and users.

Smart grids can then be seen as collective adaptive systems (CAS) in that they consist of large numbers of spatially-distributed entities that are not identical [3]–[5]. Furthermore, control is decentralised and individual entities can make individual decisions about their own behaviour. They can also communicate with other entities in the collection and this leads to adaptive behaviour. Quantitative analysis of these systems is important because modifying existing structures and infrastructure with the appropriate sensors and other hardware to be able to receive information that can then be used for decisions will involve reconfiguration costs. By being able to measure the reduction in consumption (and the associated fees), it is possible to calculate the timeframe in which the reductions in expenditure for energy consumption will cover the costs of reconfiguration. The approach taken in this paper to quantitative analysis is to build a dynamic model of the system in a quantitative process algebra, and by experimenting with this model, to develop an understanding of the reductions involved.

Modelling is a process which allows for description of the important aspects of a system while omitting aspects that are not of interest for the question under consideration. In the case of dynamic models, simulation can be used to explore the behaviour of different variants of the model, which in this context, capture different energy usage policies. A great advantage of modelling, assuming a sufficiently accurate model, is that experimentation can be done that is unlikely to be possible with the real system.

Stochastic HYPE is a quantitative process algebra [6]–[8] that allows the modelling of dynamic systems capturing events that happen immediately a condition becomes true (instantaneous behaviour), events that happen after or take a randomly distributed amount of time (discrete stochastic behaviour) and influences that affect a variable of the system that can be described by a function of time (continuous deterministic behaviour). The latter type of behaviour is described by ordinary differential equations (ODEs). The other types of behaviour are described by guards (boolean conditions) or random variables, and also allow the possibility of changing values of the system variables. The overall behaviour of a stochastic HYPE consists of trajectories of variable which are generated by the model. These trajectories consist of smooth change of variables described by ODEs, interspersed with discontinuities which occur when the system switches to a different set of ODEs, and jumps (which are also discontinuities) when the values of variables are explicitly changed.

The structure of the paper is as follows. First, the residential smart energy scenario is introduced. Next, the stochastic HYPE model is described (in limited detail, due to space constraints) highlighting important aspects of stochastic HYPE. Experimental results for the model are presented, and then a proposal for extending the model to a spatial model is given, followed by conclusions.

II. Renewable energy storage using PHEVs

The model is strongly influenced by the scenarios described in [9]–[11]. However, since the aim of the stochastic HYPE model is as an illustration of what is possible, the scenario is less complex. A more complex model will be developed as further research. Under consideration is a

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Prefix with influence: $\dfrac{}{\langle \mathrm{a} : (\iota, r, I).P, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P, \sigma[\iota \mapsto (r, I)] \rangle}$ $\mathrm{a} \in \mathcal{E}$

Prefix without influence: $\dfrac{}{\langle \mathrm{a}.P, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P, \sigma \rangle}$ $\mathrm{a} \in \mathcal{E}$

Choice: $\dfrac{\langle P, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P', \sigma' \rangle}{\langle P + Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P', \sigma' \rangle}$ $\dfrac{\langle Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle Q', \sigma' \rangle}{\langle P + Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle Q', \sigma' \rangle}$

Constant: $\dfrac{\langle P, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P', \sigma' \rangle}{\langle A, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P', \sigma' \rangle}$ $A \stackrel{\mathit{def}}{=} P$

Cooperation without synchronisation: $\dfrac{\langle P, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P', \sigma' \rangle}{\langle P \bowtie_L Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P' \bowtie_L Q, \sigma' \rangle}$ $\mathrm{a} \notin L$ $\dfrac{\langle Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle Q', \sigma' \rangle}{\langle P \bowtie_L Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P \bowtie_L Q', \sigma' \rangle}$ $\mathrm{a} \notin L$

Cooperation with synchronisation: $\dfrac{\langle P, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P', \tau \rangle \quad \langle Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle Q', \tau' \rangle}{\langle P \bowtie_L Q, \sigma \rangle \xrightarrow{\mathrm{a}} \langle P' \bowtie_L Q', \Gamma(\sigma, \tau, \tau') \rangle}$ $\mathrm{a} \in L$, Γ defined

Figure 1. Operational semantics for stochastic HYPE where E, E', F and F are stochastic HYPE models, σ, τ and τ' are states, $\mathcal{E}$ is the set of events and $L \subseteq \mathcal{E}$.

distribution transformer serving a small number of houses (between four and seven [9]) and supplying electricity to the houses. Additionally, the houses have wind turbines to generate electricity. It is assumed that each house has a PHEV (plug-in hybrid electric vehicle) with a battery. Each house has a controller that receives information about the current price of electricity, and can decide how to charge the battery, from the grid and/or the turbine. Furthermore, the controllers are networked so that they can communicate with other controllers which allows for wind energy to be shared. The adaptive aspects come from the fact that controllers react to the information they receive, and the collective aspect comes from the interaction of the different controllers. The questions to be answered about this scenario include how much is saved by using wind turbines to charge the batteries which is the focus of this paper. The model considers weekday behaviour only with the assumption that the vehicle while be absent for most of the day.

III. Stochastic HYPE model

A stochastic HYPE model has two parts – the uncontrolled system Sys which consists of subcomponents that each describe the capabilities of the system in terms of the change in the continuous behaviour specified by each subcomponent as it reacts to certain events, and controllers Con which impose ordering on events, and only deal with discrete behaviour – either instantaneous or stochastic. The initial state of the model that will be presented in this paper can be expressed as

$$SE \stackrel{\mathit{def}}{=} Sys \bowtie_{*} \underline{\mathrm{init}}.Con.$$

Here $\bowtie_{*}$ means synchronisation on all shared events. We assume a number of real-valued variables that are changing continuously in the model including B_i for the current charge of a battery, C_i for the cost of electricity from the grid consumed by a single house and T for time. Each battery can be charged from the grid or from the wind turbine when the PHEV is present and the controller that expresses this is defined as follows. It captures the four possible states of charging: *none*, *grid* charging, *wind* charging, and *both*. A fifth state occurs when the PHEV is away from the house.

$$\begin{array}{llcl}
\textit{none:} & BC_i & \stackrel{\mathit{def}}{=} & \underline{\mathrm{gch}}_i.BC_{i,1} + \underline{\mathrm{wch}}_i.BC_{i,2} \\
 & & + & \underline{\mathrm{go}}_i.\underline{\mathrm{return}}_i.BC_i \\
\textit{grid:} & BC_{i,1} & \stackrel{\mathit{def}}{=} & \underline{\mathrm{nogch}}_i.BC_i + \underline{\mathrm{wch}}_i.BC_{i,3} \\
 & & + & \underline{\mathrm{go}}_i.\underline{\mathrm{return}}_i.BC_i \\
\textit{wind:} & BC_{i,2} & \stackrel{\mathit{def}}{=} & \underline{\mathrm{nowch}}_i.BC_i + \underline{\mathrm{gch}}_i.BC_{i,3} \\
 & & + & \underline{\mathrm{go}}_i.\underline{\mathrm{return}}_i.BC_i \\
\textit{both:} & BC_{i,3} & \stackrel{\mathit{def}}{=} & \underline{\mathrm{nogch}}_i.BC_{i,2} + \underline{\mathrm{nowch}}_i.BC_{i,1} \\
 & & + & \underline{\mathrm{go}}_i.\underline{\mathrm{return}}_i.BC_i
\end{array}$$

There are two other controllers that sequence other events in the model.

$$\begin{array}{rcl}
Wind & \stackrel{\mathit{def}}{=} & \underline{\mathrm{blow}}.\underline{\mathrm{noblow}}.Wind \\
P & \stackrel{\mathit{def}}{=} & \underline{\mathrm{peak}}_d.\underline{\mathrm{midpeak}}_d.\underline{\mathrm{peak}}_e.\underline{\mathrm{midpeak}}_e.\underline{\mathrm{offpeak}}.P
\end{array}$$

The first is a simple sequencer that describes the wind starting to blow at a sufficient speed for the turbine to generate electricity and stopping. The second describes the sequence of different prices for electricity. Each term of the form $\underline{\mathrm{a}}$ in the controllers above is an instantaneous event, and controllers have the standard process algebra semantics with $\underline{\mathrm{a}}.P$ as prefix and $P_1 + P_2$ as choice. Each event $\underline{\mathrm{a}}$ has event conditions of the form

$$ec(\underline{\mathrm{a}}) \stackrel{\mathit{def}}{=} (\text{Boolean condition}, \text{change in variable values})$$

For example, the event $\underline{\mathrm{gch}}_i$ which describes charging the battery of the PHEV from the grid will have a Boolean condition that captures the policy that determines when the system should switch on charging from the grid. Likewise the Boolean condition associated with $\underline{\mathrm{nogch}}_i$ will determine when the system should switch off charging the battery from the grid. Choices for these policies will be discussed below. A more concrete example is that of the event $\underline{\mathrm{peak}}_d$. The daytime peak period starts at 07:00 and the event condition involves checking the time and changing the price of electricity from the grid.

$$ec(\underline{\mathrm{peak}}_d) \stackrel{\mathit{def}}{=} (T \,\mathit{mod}\, 24 = 7, \mathit{Gcost} = \mathit{gcost}_{\mathit{peak}})$$

The condition checks the time variable T to see whether it is 07:00 and the variable $Gcost$ which represents the cost of electricity is changed to the peak period cost. Stochastic HYPE also allows for stochastic events $\overline{\text{a}}$ where the event condition has the form

$$ec(\overline{\text{a}}) \stackrel{def}{=} (\text{functional rate}, \text{change in variable values})$$

Here the event occurs at the end of a duration drawn from the exponential distribution defined by the rate which may depend on the variables of the system. Another way to introduce stochasticity is with timers, and this is the approach taken in this model.

$$\begin{array}{lcl} ec(\underline{\text{go}}_i) & \stackrel{def}{=} & (T >= T_i, T_i = T + \gamma) \\ ec(\underline{\text{return}}_i) & \stackrel{def}{=} & (T >= T_i, T_i = T + \gamma' \wedge B_i = B_i - \beta) \end{array}$$

The values γ and γ' are obtained from random distributions that describe the pattern of the PHEV being present at the house or absent. The value β is also randomly generated and represents the decrease in batter charge while away from the house. More will be said about the actual distributions chosen for modelling in Section V. Assuming n houses and PHEVs, the controllers of the system can then be defined as follows.

$$Con \stackrel{def}{=} (BC_1 \bowtie_* \ldots \bowtie_* BC_n) \bowtie_* Wind \bowtie_* P$$

Next, the continuous part of the system must be defined. Subcomponents describe how different influences affect the variables of the system. Each subcomponent describes a specific influence consisting of an influence name and two other elements and there is a function iv which links each influence name to a system variable. The model under consideration has the following influence names.

$$iv(c_i) = C_i \quad iv(bg_i) = iv(bw_i) = B_i \quad iv(t) = T$$

The subcomponent for time has the form

$$Time \stackrel{def}{=} \underline{\text{init}} : (t, 1, 1).Time$$

which says that on the event $\underline{\text{init}}$ (which is always the first event of any simulation and occurs immediately the simulation starts), the influence t has strength 1 and is linear in form, hence the second 1. This describes that time increases with rate 1, as we would expect. No other events can affect the passing of time. The other subcomponents are less straightforward. Note that the battery variables B_i each have two influences, and a subcomponent for each influence name. One subcomponent captures the effect of charging from the grid and the other charging from the wind.

$$\begin{array}{lcl} BattG_i & \stackrel{def}{=} & \underline{\text{init}} : (bg_i, 0, 0).BattG_i \\ & + & \underline{\text{go}}_i : (bg_i, 0, 0).BattG_i \\ & + & \underline{\text{nogch}}_i : (bg_i, 0, 0).BattG_i \\ & + & \underline{\text{gch}}_i : (bg_i, ch_{max} - ch_w, 1).BattG_i \end{array}$$

The first three events mean that the battery is not charging and hence the influence name is associated with zeroes. The last event captures charging from the grid, and the rate of charge is specified to be the maximum at which the battery can charge minus the charge obtained from the wind turbine (which could be zero).

$$\begin{array}{lcl} BattW_i & \stackrel{def}{=} & \underline{\text{init}} : (bw_i, 0, 0).BattW_i \\ & + & \underline{\text{go}}_i : (bw_i, 0, 0).BattW_i \\ & + & \underline{\text{nowch}}_i : (bw_i, 0, 0).BattW_i \\ & + & \underline{\text{wch}}_i : (bw_i, ch_w, 1).BattW_i \end{array}$$

The component for charging the battery from the wind turbine is similar in structure.

$$\begin{array}{lcl} Grid_i & \stackrel{def}{=} & \underline{\text{init}} : (c_i, 0, 0).Grid_i \\ & + & \underline{\text{go}}_i : (c_i, 0, 0).Grid_i \\ & + & \underline{\text{nogch}}_i : (c_i, 0, 0).Grid_i \\ & + & \underline{\text{gch}}_i : (bc \cdot (c_i, ch_{max} - ch_w), Gcost).Grid_i \end{array}$$

This subcomponent describes the cost of electricity used for charging a battery. The first three events occur when charging is stopped and hence the influence becomes zero. The last event occurs when charging from the grid starts and this consumption incurs costs. The cost of the electricity is given by $Gcost$ as discussed above, and the maximum consumption of electricity per time unit by the battery is given by the value bc. This value is modified according to whether part of the charge is coming from the wind turbine. It is assumed that the PHEVs have batteries with identical characteristics. The uncontrolled system can now be defined by

$$\begin{array}{lcl} Sys & \stackrel{def}{=} & (BattG_1 \bowtie_* \ldots \bowtie_* BattG_n) \bowtie_* \\ & & (BattW_1 \bowtie_* \ldots \bowtie_* BattW_n) \bowtie_* \\ & & (Grid_1 \bowtie_* \ldots \bowtie_* Grid_n) \bowtie_* Time \end{array}$$

The uncontrolled system can be combined with the controllers which are prefixed by the event $\underline{\text{init}}$ to ensure it is the first event that happens to, give the full model.

$$SE \stackrel{def}{=} Sys \bowtie_* \underline{\text{init}}.Con$$

IV. Stochastic HYPE semantics

The behaviour of a HYPE is described by the values of the variables of the system over time. For each variable, its trajectory consists of periods of smooth variation in the value of the variable as defined by the current ODE for that variable punctuated with discontinuities which may include jumps in the value of the variable. These discontinuities represent a switch from one ODE for the variable to another ODE, and happens when a stochastic event completes or an instantaneous event occurs.

The structured operational semantics for stochastic HYPE are given in Figure 1. These define a transition system labelled with events over configurations. Configurations are pairs consisting of a stochastic HYPE term, and a state that keeps track on the current details for each influence. The state is a function that maps each influence name to a pair consisting of the second element (the influence strength) and third elements (the form that the influence takes which allows for the introduction of variables) of its tuple. The semantics involve two functions: the first function which appears in the first prefix rule is a straightforward update of the state. The second, Γ is more interesting as for each influence it detects what changes have occurred in σ caused by the transitions and is only defined if no influence has been changed by both transitions. In other words, the transition in the conclusion can only be inferred if each influence is only modified by one transition in the premise of the rule, or none. It can be shown that for certain types of well-defined stochastic HYPE models [6], [7], Γ is always defined and this is true for the model presented here. To illustrate the semantics, consider when one of $\underline{\text{init}}$, $\underline{\text{go}}_i$ or $\underline{\text{nogch}}_i$ has occurred. In this case, $\sigma(c_i) = (0, 0)$.

The labelled transition system describes the potential dynamics of the system. For each configuration $\langle Sys \underset{*}{\bowtie} Con, \sigma \rangle$, the state σ can be used to obtain ordinary differential equations (ODEs) which describe the continuous change over time of the system variables. For each variable V,

$$\frac{dV}{dt} = \sum \{\!| \, rI \mid iv(\iota) = V, \sigma(\iota) = (r, I) \, |\!\}$$

The ODE is defined as the sum of rI for each influence name[1] that provides an influence on V. Hence different configurations in the labelled transition system will provide different ODEs. For the state mentioned above,

$$\frac{dC_i}{dt} = 0$$

as there is no change in the consumption when the battery is not being charged. When the battery is being charged from the grid, the following ODE will be used.

$$\frac{dC_i}{dt} = bc \cdot (ch_{max} - ch_w) \cdot Gcost$$

This ODE describes how the overall costs of consumption increase per time unit by the cost per energy unit and the energy consumed. Depending on the value of ch_w, it either takes into account that there is partial charging from the wind turbine and reduces the amount that is taking from the grid, or it takes all the power required from the grid when ch_w is zero. The ODE for when the battery is being charged by both grid and wind has the form

$$\frac{dB_i}{dt} = (ch_{max} - ch_w) + ch_w = ch_{max}$$

To be able to explicitly describe the dynamics of a stochastic HYPE model, it must be mapped to an appropriate mathematical structure that describes the different types of behaviour enabled in the model formalism. Piecewise deterministic Markov processes (PDMPs) are an appropriate mathematical model [12] but their presentation is difficult to work with, hence transition-driven stochastic hybrid automata [13] are used as an intermediary. These are embodied in the stochastic hybrid simulator described in [14] and which was used for the simulations reported in this paper.

[1] The sum must be over a multiset because rI may appear more than once.

V. Simulation and analysis

The first question of interest covers the saving introduced by being able to charge the battery from the wind-turbine. We assume each householder is allowed to set a threshold h_i for their battery under which energy will be drawn from the grid to charge the battery even if it is during a peak period. This threshold ensures that a minimum amount of battery charge will be available when first driving the vehicle away from the house, and hence reduces the fuel costs of driving. While charging, once this threshold has been reached, further charging will only occur in midpeak or offpeak periods. This describes the policy when there is no wind turbine present. In the presence of the wind turbine, the policy can be modified in a number of ways, and the simplest is to allow charging from the wind turbine whenever there is sufficient wind, regardless of the cost of electricity from the grid. A more collective scenario allows for wind energy to be shared with immediate neighbours when it is not required (either because the vehicle is absent or the battery is fully charged). It is assumed that the four houses are arranged in a ring and so each householder has two immediate neighbours with whom they can share energy.

Two quantities can be defined to assess the performance of a smart energy system [11]. *Cost efficiency* is defined by the ratio of savings obtained from renewable energy to cost of energy without renewables. *Energy efficiency* is defined by the ratio of the renewable energy to all energy. *Wind efficiency* will also calculated as the ratio of wind used to generate energy versus actual wind available. All of these measures will be expressed as percentages.

The choice of parameters for the model is not straightforward. Technical specifications for the battery (capacity, time to charge, power required to charge and the battery charge used per kilometre of travel) and the wind turbine (rated power, rated speed and cut-in speed) are taken from [11]. Percentage of the time that sufficient wind is available, and average wind speed of sufficient wind have been estimated from British wind speed frequency data. The time at which to leave the house is an exponentially distributed time after 7am and time at which to return, an exponentially distributed time after 4pm. This ensures that journeys are not randomly spread over day and night, and hence the model essentially only considers weekdays. This approach also captures the fact that the PHEVs will be ready for charging during a peak period mimicking what is likely to happen in reality. Finally, distance is determined by an exponential distribution with a mean of 40 kilometres. Distance is used in the calculation of the charge of the battery on return to the house. Most of the distributions chosen for the model are based on the exponential distribution but other distributions can be used. The peak cost of electricity is £0.272 per kWh during peak periods, £0.194 per kWh during mid-peak and £0.107 per kWh during off-peak [11].

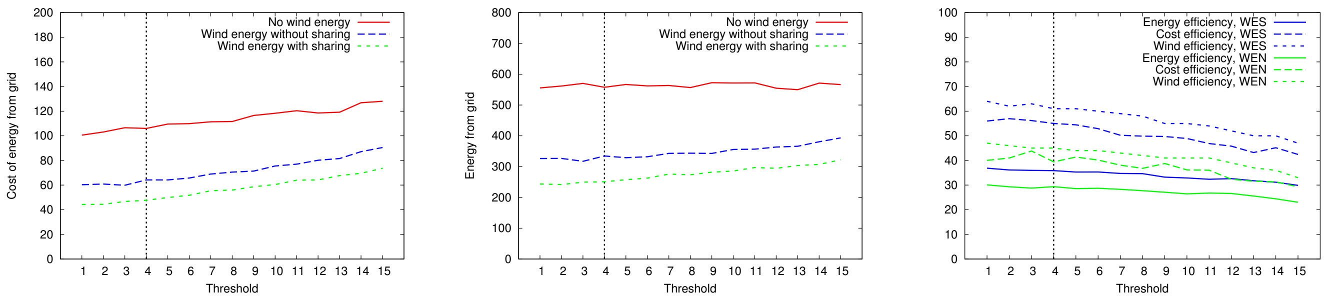

Figure 2. Comparison over different charging thresholds: grid energy cost (left), grid energy consumption (center) and efficiencies (right) (WEN: wind energy without sharing, WES: wind energy with sharing)

As mentioned above, a householder can set a threshold h_i under which the PHEV will always be charged from the grid. Setting this to 4 kWh (for a 16 kWh battery) for all households, for a simulation of 20 days (with 50 individual simulation runs), provides a average cost of £106.51 for each household without wind generation, £64.19 for each household with wind generation without sharing, and £47.67 for each household with wind generation with sharing. Thus, wind generation without sharing has a cost efficiency of 40% and wind generation with sharing has a cost efficiency of 55%. To see the impact of changing the thresholds, an experiment was performed varying the threshold value from 1 to 15, in each case with 50 simulations over a 20 day period. The tool used to run the simulation is described in [14]. The results are given in Figure 2 and the results for a threshold of 4 kWh are highlighted by the vertical dashed line. The total cost of energy increases as the threshold increases since a higher threshold allows more energy to be consumed during peak periods. The middle graph shows that energy consumption without wind energy is consistent across all thresholds but with wind energy the energy consumed from the grid increases as the threshold increases. The right hand graph show how efficiencies of all types decrease as the threshold increases. With sharing of energy and the lowest threshold, 64% of the available wind is used. The fact that the vehicles are away for most of the day means that there will be periods when all vehicles are absent and wind energy cannot be harnessed. This can be solved by a local battery which is charged from the wind energy and then used to charge the PHEV battery in the evening, or by using this wind energy for appliances that run during the day such as refrigerators and washing machines.

This model is suitable for multiple experiments, beyond considering variation of parameters. It is important not to overload the transformer that serves the group of houses, and this is most likely to happen during peak periods when many appliances are on, and PHEVs are charging their batteries. Hence, a modification of the model would be to limit the number of PHEVs that can charge during peak times, raising the question of whether this should be done on a first-come first-served basis or something more principled.

If a single large turbine is cheaper to install than four smaller turbines, then investigating the cost efficiency of such a system would be of interest. Having a single turbine able to charge any battery would have the advantage that any battery that is not full could be charged regardless of the house it is associated with, and takes the benefits of sharing once step further. However, if more than one battery needs charging, how should the energy be divided? Equally, or if during peak periods, should it go to the PHEVs with levels below the threshold? What if different householders set different thresholds? These are all questions whose answers can be investigated by experimentation with the model.

The model also lends itself to further extension. Introducing appliances into the model that use energy from the turbine or the battery when electricity prices are high would increase the usage of wind. Solar panels could also be added, as could local batteries not associated with the PHEVs.

Another dimension of potential extension is delivering power back to the grid or other groups of houses. In some cases, power providers have an agreement with consumers who generate electricity that the consumers' consumption is the difference between what they generate and what they consume (meaning that the feed-in tariff is the same as the consumption tariff). In other cases, the feed-in tariff for the electricity generated by consumers is less than the consumption tariff. In this latter case, it may make sense to exchange power with other groups of houses nearby. This will depend on the cost of the infrastructure which is likely to increase with the distance that the electricity needs to travel, making smaller and more local solutions more attractive.

VI. Future Work

The model presented here is an illustration of how smart grid systems can be modelled in the process algebra stochastic HYPE. However, this model is only an initial step in a much larger project. A major goal of this research is to investigate scalable approaches to modelling. This means that as models become larger with more and more components, it is still possible to analyse them. One approach to scalability is to use fluid or mean-field approaches where discrete stochastic behaviour based on exponential distributions can be approximated by continuous deterministic behaviour [15], and this approach has been successfully applied within process algebraic modelling [16].

In this paper, a small group of houses that are served by a single transformer has been considered. However, neighbourhoods and suburbs consist of many such groups, laid out across the landscape. The scalability question raised is whether one can efficiently model consumption and efficiency in these suburbs *including* the spatial heterogeneity that may affect renewable resources, such as different orientations of solar panels on different houses, obstacles that impede wind flow, and possible failures of equipment. The model presented in this paper is detailed, and the resources required to simulate many versions of this model within a single are impractical. Hence, the aim is to find a way to abstract from the details while obtaining good approximations. Moreover, the goal is to obtain a general solution that allows one to transform stochastic HYPE models to more scalable ones, rather than a solution that is specific to this model. Clearly, the quantities that are treated continuously such as electricity consumptions should remain continuous, and the challenge is how to treat non-continuous aspects of the model such as householder behaviour as continuous. This model also has aspects of systems-of-systems so recent work on ODEs for such models is relevant [17]. Since this model is hybrid, results relating to approximations in the presence of guards and/or instantaneous transitions are a useful place to start [18]. Additionally, spatial modelling has been considered in a number of disciples and existing approaches need to be investigated [19].

To conclude, stochastic HYPE is a suitable model for the scenario posed here, and the model can be extended in a number of interesting directions. However, this type of simulation is not scalable for multiple groups of houses, and further work involves developing techniques for such abstractions and appropriate model transformations.

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